Brit. J. Phil. Sci. **49** (1998), 49–65

Hume on Miracles: Bayesian Interpretation, Multiple Testimony, and the Existence of God

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ABSTRACT

Hume's argument concerning miracles is interpreted by making approximations to terms in Bayes's theorem. This formulation is then used to analyse the impact of multiple testimony. Individual testimonies which are 'non-miraculous' in Hume's sense can in principle be accumulated to yield a high probability both for the occurrence of a single miracle and for the occurrence of at least one of a set of miracles. Conditions are given under which testimony for miracles may provide support for the existence of God.

1 Introduction

A number of authors over the last few years have considered Hume's celebrated essay on miracles[1] from a Bayesian perspective. The bulk of this discussion has centred around the issues of whether Hume can rightly be considered a proto-Bayesian, how exactly to interpret him in terms of the calculus of probabilities, and hence how to judge whether his argument is correct. There has been some further discussion of whether testimony for miracles can provide evidence for the existence of God.

An important point for further consideration is the impact on the argument of multiple testimony, understood either in the sense of several independent testimonies for a single miracle or of independent testimonies for a number of miracles. In the relatively recent literature Mackie[2] has noted the strong

[1] Hume [1748]. [2] Mackie [1982], pp. 25–6.

evidential force of two independent testimonies for a single miracle, and Earman[3] has provided some analysis of this from the perspective of the probability calculus. Also, Sorensen[4] has noted the possibility that combined testimony for many miracles may yield a high probability that at least one has occurred. However, this latter claim has been dismissed by Schlesinger[5] on the grounds that the occurrence of one miracle is not independent of the occurrence of any other. This latter assumption requires further examination.

Both Schlesinger and Swinburne[6] have argued that testimony for miracles provides evidence for the existence of God. Otte[7] has challenged Schlesinger's version of the argument. Again, this topic warrants further consideration.

This paper first establishes the Bayesian procedure to be followed, briefly noting some of the related work done elsewhere, before proceeding to examine the issues of multiple testimony and whether testimony for miracles provides evidence for the existence of God.

2 Bayesian framework

The critical passage from Hume's *Enquiry* states:

> The plain consequence is (and it is a general maxim worthy of our attention), 'That no testimony is sufficient to establish a miracle, unless the testimony be of such a kind, that its falsehood would be more miraculous, than the fact, which it endeavours to establish; and even in that case there is a mutual destruction of arguments, and the superior only gives us an assurance suitable to that degree of force, which remains, after deducting the inferior.'[8]

On the face of it, Hume is distinctly non-Bayesian since he speaks of subtracting probabilities. In the case when the miracle and the falsehood of the testimony to it are equally miraculous, which we interpret to mean 'have equal but very low probabilities', Hume would ostensibly give zero probability to the miracle given the testimony since the a priori probability of the miracle and the probability that the testimony is false cancel out. This interpretation is clearly incorrect since it implies that 'miraculously' reliable testimony has reduced rather than increased the probability of the miracle's occurrence.

Hume's *Enquiry* was published in 1748 and Bayes's theorem was published posthumously in 1763, so naïvely one would expect Hume to be ignorant of the latter. However, the more important question is not the historical but the philosophical one, namely 'Can Hume's argument be rephrased in Bayesian terms?'

On a Bayesian interpretation, one would expect, in the case of equally 'miraculous' miracle and falsehood of testimony, that the probability of the miracle having occurred given the testimony would be 0.5. In this case one is

[3] Earman [1993]. [4] Sorensen [1983]. [5] Schlesinger [1987].
[6] Swinburne [1991], Ch. 12. [7] Otte [1993]. [8] Hume, *op. cit.*, pp. 115–16.

indifferent between the miracle and the falsehood of the testimony.[9] In order to make the miracle rationally acceptable, one requires the probability of the miracle having occurred given the testimony to be greater than the indifference level of 0.5, and for this to entail, on Hume's reasoning, the probability that the testimony is false to be less than the *a priori* probability of the miracle's occurrence.

Following Hume, we define a miracle to be 'a violation of the laws of nature'.[10] It is true that Hume modified this simple definition in a footnote: 'A miracle may be accurately defined, *a transgression of a law of nature by a particular volition of the Deity, or by the interposition of some invisible agent.*'[11] This confuses the issue: it is one thing to receive testimony to an event that may be deemed to constitute a violation of a law of nature, and quite another to assess the likelihood of a divine origin of the event (see Earman).

We develop the Bayesian framework as follows.

Let M denote a specific miracle occurring at space-time location $\mathcal{L}$.
Let K denote background knowledge that a certain witness W was at $\mathcal{L}$ and made a report about what happened at $\mathcal{L}$.
Let T be testimony to M provided by W, i.e. T is W's assertion that M occurred.

Then we define the following probabilities:

Let
P[M] = *a priori* probability of the occurrence of M,
P[M|T] = probability that M has occurred given T,
P[T|M] = probability that T is provided given that M has indeed occurred,
P[T|~M] = probability that T is provided given that M has not occurred.

All probabilities are further conditioned on K, though this dependence is omitted for convenience. M might be, to take Schlesinger's example: 'the walls of Jericho collapsed upon the blowing of the Israelite trumpets.'[12] T would be the statement of a witness W, at Jericho at the time, that M has occurred.

With this notation, Bayes's theorem states:

$$P[M|T] = \frac{P[T|M]\,P[M]}{P[T|M]\,P[M] + P[T|\sim M]\,P[\sim M]} \tag{1}$$

Now a miracle is, as Hume asserts, intrinsically improbable. If we also assume that W is highly reliable, then the following approximations and inequalities hold for the probabilities entering equation (1):

$P[T|M] \simeq 1$
$P[M] \ll 1$
$P[T|\sim M] \ll 1$
$P[\sim M] \simeq 1$

[9] Owen [1989]. [10] Hume, *op. cit.*, p. 114. [11] *Ibid.*, footnote, p. 115.
[12] Schlesinger, *op. cit.*, p. 224.

With these assumptions we obtain the approximate relationship

$$P[M|T] \simeq \frac{P[M]}{P[M] + P[T|{\sim}M]} \tag{2}$$

This latter expression is greater than or less than 0.5 as P[M] is greater than or less than P[T|~M]. Our theorem can be stated succinctly thus (always implicitly assuming that P[M] > 0):

$$P[M|T] \gtrsim 0.5 \leftrightarrow P[M] \gtrsim P[T|{\sim}M] \tag{3}$$

where the symbol '$\gtrsim$' is used to indicate 'greater than or approximately equal to'.

Other authors provide interpretations of, or advances on, Hume which are broadly equivalent to the above, especially when the régime of values taken by the probabilities is taken into account (see, for example, Owen,[13] Dawid and Gillies,[14] Schlesinger,[15] Sobel,[16] Millican,[17] and Earman[18]).

Whether or not Hume meant this, and there is some dispute among the authors cited, the comparison between P[T|~M] and P[M] seems to me the most natural to make, having a good rationale, and we shall proceed on this basis. As a convenient shorthand we shall denote testimony for which P[M] $\gtrsim$ P[T|~M] as 'miraculous' and testimony for which P[M] < P[T|~M] as 'non-miraculous', though clearly the terms are relative to the miracle in question; in Hume's terms the former would be 'testimony the falsehood of which would be more miraculous than the miracle which it endeavours to establish'.

Before examining the impact of multiple testimony, we note that Schlesinger has provided a powerful counter to Hume's argument in the single testimony case.[19] Let us suppose, for the sake of argument, that W tells the truth about any matter with probability t. It follows that $P[T|M] = t$.

Now Owen argues that $P[T|{\sim}M] = 1 - t$. But the matter is not quite so simple, at least given the formulation here, which we have tried to make as careful as possible. Remember that our background knowledge K is that W is in a position to make a report on what occurs and does so. T is, specifically, testimony for M, i.e. 'W states that M occurred'. But given that M did not occur there are many ways for W to give a false report (either deliberately to lie or to be mistaken), and it is most unlikely that the false report W would come up with is M. In fact

$$\begin{aligned} P[T|{\sim}M] &= P[\text{W gives a false report}] \\ &\quad \times P[\text{the false report W gives is T, i.e. testimony for M}] \\ &= (1 - t) \times (1/n) \end{aligned} \tag{4}$$

[13] Owen, *op. cit.* [14] Dawid and Gillies [1989]. [15] Schlesinger, *op. cit.*
[16] Sobel [1987, 1991]. [17] Millican [1993]. [18] Earman, *op. cit.* [19] Schlesinger [1991].

where n is the very large number of false reports to choose from (assuming for the sake of argument that these are equally probable). Hence the comparison is not just between the improbability of the miracle and the unreliability of the witness, but also depends on the probability that the witness would specifically come up with M, given that M did not occur. When the latter is taken into account, the posterior probability of the miracle may well exceed 0.5.

3 Independent testimonies to a single miracle

A point that seems to have been overlooked in the discussion of Owen and Sobel, but noted by Earman and Sorensen, is that of multiple testimony to miracles. If we have several independent testimonies to a miracle, or independent testimonies to different miracles, it can be the case that the probability that a miracle has occurred is greater than 0.5, even though no testimony considered individually was as miraculous as the miracle.

Earman has noted that, with a sufficient number of independent witnesses, one could establish any miracle as effectively certain. If T^n is taken to mean that there are n independent witnesses, and for simplicity we assume that they each testify truly with probability $p = \mathrm{P[T|M]}$ and falsely with probability $q = \mathrm{P[T|{\sim}M]}$, then, assuming independence, we obtain

$$\mathrm{P[M|T^n]} = \frac{\mathrm{P[T^n|M]\;P[M]}}{\mathrm{P[T^n|M]\,P[M] + P[T^n|{\sim}M]\;P[{\sim}M]}}$$

$$= \frac{\mathrm{P[T|M]^n\;P[M]}}{\mathrm{P[T|M]^n\;P[M] + P[T|{\sim}M]^n\;P[{\sim}M]}} \quad \text{(utilizing the independence condition)}$$

$$= \frac{1}{1 + \left[\frac{\mathrm{P[{\sim}M]}}{\mathrm{P[M]}}\right]\left[\frac{q}{p}\right]^n} \tag{5}$$

As $n \rightarrow \infty$, $(q/p)^n \rightarrow 0$ and so $\mathrm{P[M|T^n]} \rightarrow 1$. More significantly, as Earman points out, no matter how small P[M] one can choose n (finite) such that $\mathrm{P[M|T^n]} > 0.5$. The point is that, as n increases, the factor $(q/p)^n$ (which arises because of the multiplicative law for combining independent probabilities) decreases very rapidly, soon becoming comparable with P[M].

Whether or not there have been examples in history where the combined testimony of independent witnesses has enhanced the probability of a miracle's occurrence to greater than indifference level is a matter for empirical investigation. The point is that Hume's (already questionable) assumption that no individual testimony can be this reliable does not preclude the combined testimonies being so.

4 The aggregation of testimonies to several miracles

Now let us consider the case of independent testimonies for two miracles. Suppose specifically that we have independent testimonies T_1 and T_2 for miracles M_1 and M_2 respectively.

Now one might naïvely suppose that we can combine the probabilities for miracles M_1 and M_2 occurring given testimonies T_1 and T_2 as follows:

Probability that at least one of M1 and M2 occurs = 1 − Probability that neitherM_1 nor M_2 occurs

= 1 − (Probability that M_1 does not occur)

× (Probability that M_2 does not occur)

i.e. using the notation of the probability calculus,

$$\begin{aligned} P[M_1 \vee M_2 | T_1 \wedge T_2] &= 1 - (1 - P[M_1 | T_1])\,(1 - P[M_2 | T_2]) \\ &= P[M_1 | T_1] + P[M_2 | T_2] \\ &\quad - P[M_1 | T_1]\, P[M_2 | T_2] \end{aligned} \tag{6}$$

One can imagine, given that there are very many reported miracles, iterating equation (6) to build up a substantial probability that at least one occurred. The occurrence of at least one miracle might well become probable.

Sorensen notes that Hume has at best established a case by case scepticism. The wise man will not believe any testimony that a specific miracle took place. However, this does not rule out the wise man, equally rationally, believing that one of a set of reported miracles, though it is not known which, did actually occur. If the analysis in the previous section is correct, then case-by-case scepticism is also refuted on the grounds of multiple testimony, but even if this is not so, testimony for many miracles may lead one rationally to accept that at least one occurred.

Schlesinger has pointed out a flaw in this argument, namely that it assumes that M_1 and M_2 are independent. Schlesinger writes the formula for combining probabilities as

$$\begin{aligned} P[M_1 \vee M_2] &= P[M_1] + P[M_2] - P[M_1 \wedge M_2] \\ &= P[M_1] + P[M_2] - P[M_2 | M_1]\, P[M_1] \end{aligned} \tag{7}$$

In the independence case $P[M_2|M_1] = P[M_2]$, but Schlesinger argues that $P[M_2|M_1] \simeq 1$. He does so on the grounds that if we know that one miracle has occurred then our reasoning to the intrinsic improbability of miracles in general is wrong, and we should instead assume that they are likely. Surely, however, they are not dependent to the extent which Schlesinger suggests. The known occurrence of a turning of water into wine at some point in space and time does not license me to believe in a levitation at some other location in space and time—even if I have testimony to the latter. Miracles do not all stand or fall together.

This discussion should more accurately proceed on the basis of probabilities conditioned on the evidence of testimony (this is implicit in Schlesinger's discussion, but needs to be made explicit). What we are really interested in is the quantity $P[M_1 \vee M_2 | T_1 \wedge T_2]$, i.e. the probability that at least one miracle has occurred given the testimony for each.

The correct, full formula for combining the probabilities is:

$$\begin{aligned} P[M_1 \vee M_2 | T_1 \wedge T_2] &= P[M_1 | T_1 \wedge T_2] + P[M_2 | T_1 \wedge T_2] \\ &\quad - P[M_1 \wedge M_2 | T_1 \wedge T_2] \\ &= P[M_1 | T_1 \wedge T_2] + P[M_2 | T_1 \wedge T_2] \\ &\quad - P[M_2 | M_1 \wedge T_1 \wedge T_2]\ P[M_1 | T_1 \wedge T_2] \end{aligned} \tag{8}$$

We make the following observations with regard to this equation:

(i) The occurrence of M_1 may well be evidence for M_2, and vice versa, as Schlesinger suggests. Hence we should retain the $M_2|M_1$ dependency.

(ii) T_2 provides evidence for M_1 indirectly through the possible occurrence of M_2. However, it is surely safe to assume that this evidence is far weaker than T_1. The same comment applies with indices reversed.

With these assumptions (8) can be approximated to obtain

$$\begin{aligned} P[M_1 \vee M_2 | T_1 \wedge T_2] &= P[M_1 | T_1] + P[M_2 | T_2] \\ &\quad - P[M_2 | M_1 \wedge T_2]\ P[M_1 | T_1] \end{aligned} \tag{9}$$

The key factor to calculate in (9) is $P[M_2 | M_1 \wedge T_2]$. In the case of independence this is simply $P[M_2 | T_2]$, and the formula reduces to that of (6) above. We agree with Schlesinger that this is too simplistic, but disagree with him that it should be taken as unity. Both M_1 and T_2 are evidence for M_2. $P[M_2 | M_1 \wedge T_2]$ can be expanded using Bayes's theorem as follows:

$$P[M_2 | M_1 \wedge T_2] = \frac{P[T_2 | M_1 \wedge M_2]\ P[M_2 | M_1]}{P[T_2 | M_1 \wedge M_2]\ P[M_2 | M_1] + P[T_2 | M_1 \wedge {\sim} M_2]\ P[{\sim} M_2 | M_1]} \tag{10}$$

We can surely make the further simplifying assumption that the occurrence of M_1 has no bearing on testimony T_2 for M_2, given that M_2 has not occurred. Then, with similar assumptions about the reliability of testimony as were made in our earlier discussion, we can approximate the above thus:

$$P[M_2 | M_1 \wedge T_2] = \frac{P[M_2 | M_1]}{P[M_2 | M_1] + P[T_2 | {\sim} M_2]\ P[{\sim} M_2 | M_1]} \tag{11}$$

Now if $P[M_2|M_1] \gg P[T_2|\sim M_2]\ P[\sim M_2|M_1]$, then $P[M_2|M_1 \wedge T_2] \simeq 1$. This would appear to be Schlesinger's position. From this it would follow that the occurrence of at least one miracle is no more likely than the occurrence of a single one.

Now we can agree that the occurrence of M_1 makes M_2 more likely because we no longer have near absolute scepticism about miracles in general, i.e. we would argue that $P[M_2|M_1] \gg P[M_2]$. However, as noted above, the occurrence of a turning of water into wine somewhere will not make the occurrence of a levitation elsewhere by any means probable; it will merely remove the prejudice we had that the latter was as intrinsically unlikely as anything could be. Thus we should still expect $P[M_2|M_1] \ll 1$. From this it follows that $P[\sim M_2|M_1] = 1 - P[M_2|M_1] \simeq 1$, and hence

$$P[M_2|M_1 \wedge T_2] \simeq \frac{P[M_2|M_1]}{P[M_2|M_1] + P[T_2|\sim M_2]} \tag{12}$$

The value of $P[M_2|M_1 \wedge T_2]$ then depends on the relative magnitudes of $P[M_2|M_1]$ and $P[T_2|\sim M_2]$, *both* of which are likely to be small numbers.

Let us take sample values as follows:

$$P[M_2] = P[M_1] = 10^{-6}$$

$$P[T_2|\sim M_2] = P[T_1|\sim M_1] = 10^{-3}$$

$$P[M_2|M_1] = 10^{-4}$$

Then

$$P[M_2|T_1] = P[M_1|T_1] \simeq 10^{-3} \quad \text{and} \quad P[M_2|M_1 \wedge T_2] \simeq 0.09$$

Hence

$$P[M_1 \vee M_2|T_1 \wedge T_2] \simeq 10^{-3} + 10^{-3} - 0.09 \times 10^{-3} = 1.91 \times 10^{-3}$$

Thus, the probability that at least one miracle occurred is indeed greater than the probability that either miracle occurred individually, contra Schlesinger (N.B. formula (6) would have given 1.999×10^{-3}).

Of course the above analysis can in principle be extended to more than two independent testimonies. Noting that there are very many reports of miracles down the ages, we suggest that it may well be the case that the combined force of the testimony for individual miracles leads to the probable occurrence of at least one miracle.

5 Impact of Bayesian formulation on Hume's argument

If Hume's argument is to be retrieved in the light of the above, then he must be interpreted to mean

(i) for any particular miracle and single testimony to that miracle, the probability that the witness gives a false report, and, of all the possibilities to choose from, gives the false report that the miracle in question occurred, is always greater than the *a priori* probability of the miracle's occurrence; and

(ii) the combined independent testimony for any single miracle cannot yield a probability at indifference level or greater for that miracle's acceptance; and

(iii) all the evidence for all miracles when combined cannot yield a probability at indifference level or greater that at least one miracle occurred.

A major charge levelled against Hume's argument, pressed for example by Armstrong,[20] is that he is begging the question. If we are to impose upon Hume the interpretations (i) to (iii) above, it seems to me that the force of the charge becomes overwhelming. But can Hume possibly mean this? The following passage from Part II of *Enquiry*, X, is relevant here:

> there is not to be found, in all history, any miracle attested by a sufficient number of men, of such unquestioned good sense, education, and learning, as to secure us against all delusion in themselves; of such undoubted integrity, as to place them beyond all suspicion of any design to deceive others; of such credit and reputation in the eyes of mankind, as to have a great deal to lose in case of their being detected in any falsehood; and at the same time, attesting facts, performed in such a public manner, and in so celebrated a part of the world, as to render the detection unavoidable: All which circumstances are requisite to give us a full assurance in the testimony of men.[21]

This passage does seem to consider the possibility of combined testimony from 'a sufficient number of men' for a single miracle, but rules out the idea that such combined testimony has ever existed. However, this passage does seem to be particularly question-begging, dismissing the possibility by fiat. Moreover it does not consider either the first possibility noted above, i.e. that the witnesses would all come up with the miracle in question, nor the third possibility, i.e. that non-miraculous testimonies for different miracles might result in a high probability that at least one of them has occurred.

As Mackie, who approves of the criteria put forward by Hume in this passage, notes,[22] it is a matter of controversy whether they have ever been met. In fact Hume cites a number of examples of attested miracles, but does not attempt to provide the detailed examination of their circumstances of occurrence which would be required to establish his claim. It should be noted that I am interpreting the above passage, and Hume's essay in general, as

[20] Armstrong [1992]. [21] Hume, *op. cit.*, pp. 116–17. [22] Mackie, *op. cit.*, p. 14.

asserting only that eyewitness testimony has never in actual fact been sufficient to establish the credibility of a miracle, rather than the stronger claim that it is impossible in principle that it do so. Whilst the general tenor of Hume's essay would seem to indicate that he means the former, on occasions he somewhat confusedly seems to lapse into the latter, e.g. in dismissing the hypothetical resurrection of Queen Elizabeth I. (N.B. he does not discuss the more pertinent example of the resurrection of Jesus.)[23] By indicating how the falsehood of a single testimony should be interpreted, and how many testimonies should be combined, the Bayesian analysis shows the stronger claim to be false and seriously undermines the weaker claim.

While agreeing with Hume's general principle that one should weigh the unlikelihood of the miracle against the unlikelihood that the witness is mistaken or dishonest, Mackie adds the rider that 'the likelihood or unlikelihood, the epistemic probability or improbability, is always relative to some body of information, and may change if additional information comes in.'[24] Just so. One may have 'non-miraculous' evidence for a miracle which yields a probability for the miracle's occurrence of less than 0.5. Further non-miraculous evidence may arise which when combined with the existing evidence raises that probability above 0.5. Alternatively, as Mackie observes, one may start with two independent witnesses whose agreement that a particular miracle has occurred is hard to explain unless the miracle has indeed occurred. These points are the stronger for the realisation of how the probabilities combine. The probability that a number of reliable independent witnesses are all mistaken or dishonest is the product of the individual probabilities that each is wrong, and this will in general be very small.

Now consider the third possibility suggested here, i.e. that the combined evidence for all miracles might yield a probability greater than 0.5 for the occurrence of at least one (even though none individually is shown to be probable, despite the above). Hume's argument is further undermined since the rational acceptance of miracles would then be established. Sorensen was correct in noting that, in order to refute all argument from miracles, Hume needs to establish a much greater scepticism than merely showing that the correctness of any report of a miracle is improbable.

The above would seem to indicate that multiple testimony poses a severe challenge to Hume's argument, especially if combined with Schlesinger's point about single testimony. The combined force of the point about single testimony, and the points regarding multiple testimony for a single miracle and testimony for several miracles, is to produce a strong cumulative case against Hume. But there is still the further question which must be addressed,

[23] Slupik [1995] argues that Hume's essay can be read self-consistently as making only the weaker claim.

[24] Mackie, *op. cit.*, p. 23.

regarding the importance of testimony to miracles—does testimony for miracles provide evidence for the existence of God? This we now address.

6 Does testimony for miracles provide evidence for the existence of God?

A number of authors have argued that testimony for miracles does indeed provide evidence for the existence of God. Here we are using the expression 'provides evidence for' in the sense of 'supports' or 'confirms'. Mathematically we require $P[G|T] > P[G]$, where $P[G]$ is the *a priori* probability of God's existence and $P[G|T]$ is the probability that God exists given testimony T for miracle M. Note that this is a much weaker requirement than making the existence of God probable, when we would require $P[G|T] > 0.5$.

Swinburne argues that testimony increases the probability that a miracle, defined as a violation of a natural law, occurred. The occurrence of a miracle in turn increases the probability that God exists. Hence testimony for miracles increases the probability that God exists. More precisely, Swinburne states:

> Certainly witness-reports can add to the probability that a violation occurred and so add to the probability that there is something not to be explained by natural processes. If *e* is merely witness-reports that a violation occurred, which are substantial evidence that a violation did occur (because the occurrence of the reports is not easily to be explained in some other way, e.g. in terms of the witnesses being misled by some non-miraculous phenomena), then *e* is more to be expected if a violation did occur that if it did not, and so marginally more to be expected if there is a God capable of bringing about violations than if there is not.[25]

Swinburne does not spell out this argument in probability calculus terms. However, we might construct a probabilistic version of the argument as follows, where we substitute T for *e* in keeping with our own notation, and write G for the hypothesis that God exists:

		(i)	$P[T|M] > P[T|{\sim}M] \leftrightarrow P[M|T] > P[M]$
		(ii)	Suppose we have T such that $P[T|M] > P[T|{\sim}M]$
(i) and (ii)	$\rightarrow$	(iii)	$P[M|T] > P[M]$ (i.e. T provides evidence for M)
		(iv)	Suppose similarly that M is such that $P[G|M] > P[G]$ (i.e. M provides evidence for G)
(iii) and (iv)	$\rightarrow$	(v)	$P[T|G] \gtrsim P[T|{\sim}G]$
(v)	$\rightarrow$	(vi)	$P[G|T] \gtrsim P[G]$

with '$\gtrsim$' now representing 'marginally greater than'.

Statement (i), and hence the step from (i) and (ii) to (iii), is uncontroversial, following straightforwardly from Bayes's theorem. These lines merely express

[25] Swinburne, *op. cit.*, p. 234.

the fact that testimony enhances the probability of a miracle. Statement (iv) is a similar expression asserting that M is a miracle which enhances the probability that God exists. The step from (iii) and (iv) to (v), and hence (vi), is the one which is not obvious, and requires a more careful analysis, as given below.

Schlesinger[26] utilizes Bayes's theorem in an attempt to show that testimony for miracles is evidence for the existence of God. He begins with Bayes's theorem in the form

$$P[G|T] = \frac{P[M|T]\, P[G|M \wedge T]}{P[M|G \wedge T]} \tag{13}$$

where again we have modified his notation to be consistent with ours. By replacing T with ~T in the above and dividing the two equations obtained one derives the following:

$$\frac{P[G|T]}{P[G|{\sim}T]} = \frac{P[M|T]}{P[M|{\sim}T]} \cdot \frac{P[G|M \wedge T]}{P[G|M \wedge {\sim}T]} \cdot \frac{P[M|G \wedge {\sim}T]}{P[M|G \wedge T]} \tag{14}$$

Schlesinger argues that the first ratio here is greater than 1, the second may be taken as equal to 1, and the third may also be taken as 1 (for the reasons for these judgements, see Schlesinger's paper).

With Otte[27] I agree that the first two ratios are as Schlesinger describes them (and I shall say a little more about this shortly). However, Otte has also rightly pointed out that, contrary to Schlesinger's argument, the third ratio here cannot necessarily be equated to unity. Indeed one would expect this last ratio to be less than 1. I am much more likely to believe that the Red Sea parted if God exists and I have testimony to the event, than if God exists and I have no such testimony. Even if the miracle we are talking about is the resurrection of Jesus, and G is the Christian God, testimony will still make a difference to our degree of belief in the miracle's occurrence. After all, no one claims that Moses, for example, rose from the dead, so my belief in God does not warrant belief in Moses' resurrection apart from testimony. Moreover, Christians tend to argue that Jesus' resurrection vindicates (provides evidence for?) the unique claims he makes about his relationship to God. To argue that the probability of this miracle's occurrence is dependent only on the existence of God and independent of testimony is insufficient: background knowledge such as Jesus' ontological status as Son of God would also be required. Schlesinger acknowledges that at the time of the miracle it is necessary that 'circumstances characteristic to those, which from a religious point of view demand the occurrence of a miracle, obtained.'[28] The problem is that arguably such circumstances did obtain both for Moses and Jesus. There is an analogy here with our discussion of the probability of the occurrence of one miracle given the occurrence of another. The existence of God would remove our prejudice that miracles were

[26] Schlesinger [1987]. [27] Otte, *op. cit.* [28] Schlesinger, *op. cit.*, p. 232.

as unlikely as anything could be, but would not license our belief in any particular miracle apart from testimony.

As with Swinburne's argument, we need a more careful analysis of this question, which is akin to our discussion above of testimony for many miracles. Bayes's theorem will again provide the necessary tool.

Essentially, we wish to express P[G|T] in terms of quantities we know or can derive: P[M|T] and P[G|M]. We begin from Bayes's theorem in the same form that Schlesinger has it:

$$P[G|T] = \frac{P[M|T]\ P[G|M \wedge T]}{P[M|G \wedge T]} \tag{15}$$

We utilize reasoning similar to that of our discussion of testimony for many miracles. Thus P[G|M ∧ T] can be approximated by P[G|M]. One way of seeing this is to write:

$$P[G|M \wedge T] = \frac{P[G \wedge T|M]}{P[T|M]} \simeq \frac{P[G|M]\, P[T|M]}{P[T|M]} \tag{16}$$

on the assumption that G and T are independent given M.

The key factor to evaluate is P[M|G ∧ T]. The occurrence of M will depend in general on both G and T (as M_2 depended on M_1 and T_2 above):

$$P[M|G \wedge T] = \frac{P[T|M \wedge G]\, P[M|G]}{P[T|M \wedge G]\, P[M|G] + P[T|{\sim}M \wedge G]\, P[{\sim}M|G]}$$

$$\simeq \frac{P[M|G]}{P[M|G] + P[T|{\sim}M]} \tag{17}$$

The approximation here depends on the following three assumptions:

(i) $P[T|M \wedge G] \approx 1$

P[T|M ∧ G] is virtually identical to P[T|M]—if the miracle has occurred, testimony is near certain, regardless of whether there is a God or not.

(ii) $P[{\sim}M|G] \approx 1$

P[~M|G] = 1 − P[M|G], and we assume that the probability of a miracle's occurrence given God, P[M|G], is higher than its *a priori* probability P[M], but still not particularly high, testimony still being required to maximise the probability. The argument is analogous to that about two miracles in Section **4**.

Of course, arguably P[M|G] might be high in some specific case like the resurrection of Jesus, with G the Christian God, although I do not think this is the case for reasons advanced earlier. If it were the case it would invalidate the particular approximation here, but strengthen the overall argument that T provides evidence for G.

(iii) $P[T|{\sim}M \wedge G] \simeq P[T|{\sim}M]$

It is whether a miracle has occurred or not which is the dominant factor in determining whether there is testimony, not the existence of God. This assumption can be seen alternatively by noting that

$$P[T|\sim M \wedge G] = \frac{P[T \wedge G|\sim M]}{P[G|\sim M]} \simeq \frac{P[T|\sim M]\,P[G|\sim M]}{P[G|\sim M]} = P[T|\sim M] \quad (18)$$

on the assumption that, given no miracle, the occurrence of testimony and the existence of God are independent.

Substituting from (17) into (15), we obtain

$$\begin{aligned} P[G|T] &\simeq P[M|T]\,P[G|M] \cdot \frac{P[M|G] + P[T|\sim M]}{P[M|G]} \qquad (19) \\ &= \frac{P[T|M]\,P[M]}{P[T|M]\,P[M] + P[T|\sim M]\,P[\sim M]} \cdot \frac{P[M|G]\,P[G]}{P[M]} \\ &\quad \cdot \frac{P[M|G] + P[T|\sim M]}{P[M|G]} \\ &\simeq \frac{P[M]}{P[M] + P[T|\sim M]} \cdot \frac{P[M|G]\,P[G]}{P[M]} \cdot \frac{P[M|G] + P[T|\sim M]}{P[M|G]} \end{aligned}$$

i.e.

$$P[G|T] = P[G] \cdot \frac{P[M|G] + P[T|\sim M]}{P[M] + P[T|\sim M]} \quad (20)$$

We are now in a position to write down the condition that testimony T for miracle M provides evidence for the existence of God, in the sense of making the hypothesis G more probable than its *a priori* value, i.e. $P[G|M] > P[G]$. This condition is

$$\frac{P[M|G] + P[T|\sim M]}{P[M] + P[T|\sim M]} > 1 \quad (21)$$

This condition is of course always met. However, it turns out that Swinburne is correct in asserting that T may only provide marginal evidence for G over $\sim$G. All depends on the relative values of $P[M]$, $P[M|G]$ and $P[T|\sim M]$.

If T is very good testimony for M, eg it represents the combined testimony $T_1 \wedge T_2$ of independent witnesses with parameters such that $P[M] \gg P[T|\sim M]$, then condition (21) will approximate to

$$\frac{P[M|G]}{P[M]} > 1 \quad (22)$$

In fact, it may well be that $P[M|G] \gg P[M]$, so that T (which is virtually equivalent to asserting that M has occurred) will be very good evidence for G in this case.

On the other hand, if we have only singular, non-miraculous testimony T for

M, such that $P[M] \ll P[T|\sim M]$, and furthermore $P[M|G] \ll P[T|\sim M]$, then $P[T|\sim M]$ dominates both numerator and denominator of (21), and T provides virtually no evidence for G.

There is a plausible in-between case in which $P[M] \ll P[T|\sim M]$, but $P[M|G] \gg P[T|\sim M]$. In that case the testimony for the miracle, though not strong (in the sense of overcoming the *a priori* improbability of the miracle), would nevertheless provide good evidence for G because if the miracle did occur it would very greatly enhance the probability of G. In this case we would have

$$P[G|T] \simeq P[G] \cdot \frac{P[M|G]}{P[T|\sim M]} \tag{23}$$

For example, if we had

$$P[M] = 10^{-6}$$

$$P[T|\sim M] = 10^{-3}$$

$$P[G] = 10^{-8}$$

$$P[M|G] = 10^{-1}$$ (N.B. remember that we have assumed that $P[M|G] \ll 1$)

then we should obtain

$$P[G|T] \simeq 10^{-6},$$

a significant enhancement over the *a priori* value P[G].

Earman asserts, by analogy with his analysis of multiple testimony to a single miracle, that the existence of God can be made probable by accumulating testimony to sufficiently many miracles. This will be the case if the occurrences of the miracles can be treated with a degree of independence, as they plausibly could be in the light of the above critique of Schlesinger's argument. Note that Earman has also assumed that each testimony to a miracle is substantial evidence for the existence of God, in the light of the multiple testimony argument for individual miracles (he assumes that $P[T|\sim M] \ll P[M]$ for each M so that condition (21) reduces to (22)).

7 Summary

As noted by a number of authors, Hume's argument about miracles can be well formulated in terms of Bayesian probability theory. The comparison between the quantities P[M] and $P[T|\sim M]$ seems to be the most useful, and what most authors use in practice. Schlesinger has noted what others have missed, namely that $P[T|\sim M]$ should be treated as a product: when this is done Hume's argument from single testimony is seriously weakened.

The possibility of multiple testimony further significantly affects Hume's argument, both in the case of a single miracle and that of independent testimony for many miracles. Earman's argument that multiple testimony for a single miracle could make the miracle probable is supported. The argument that independent testimonies to many miracles could show that at least one probably occurred, put forward by Sorensen, has been challenged by Schlesinger. The latter argued that miracles are not independent. Whilst this may well be true, it has been shown here that this is not sufficient to invalidate the argument. Provided miracles are not totally dependent, i.e. all stand or fall together regardless of the weight of testimony to them, the probability that at least one has occurred increases as the number of miracles considered increases. In principle this could make the occurrence of at least one probable, though which had occurred would remain unknown.

We suggest that the above considerations amount to a strong cumulative case against Hume's argument.

Finally, conditions under which testimony for a miracle might provide evidence for the existence of God were explored in somewhat more detail than by other authors. Testimony for a miracle would provide good evidence for the existence of God if either of two conditions hold. The first of these is that the testimony is very strong, for example it represents the combined testimony of a number of independent witnesses (thus overcoming the inherent improbability of the miracle's occurrence). The second is that, although the testimony were still relatively weak, and did not overcome the inherent improbability of the miracle attested to, the miracle would, if it occurred, very significantly enhance the probability of God's existence (to be precise: the probability of the miracle's occurrence given the existence of God would have to be greater than the probability that the witness would testify to the miracle if it had not occurred, which in turn would be greater than the *a priori* probability of the miracle). Either way, the accumulation of the testimony to many miracles could in principle make the existence of God probable, if the miracles are treated as occurring with a suitable degree of independence.

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